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Weakly Disposable Formulations of Environmental Technology in Nonparametric Production Analysis

Introduction

The variable returns to scale (VRS) based formulation of weakly disposable (WD) environmental technology is described in the nonparametric data envelopment analysis (DEA) literature in two alternative ways: either by a constant (uniform) abatement factor,[1] or by differential (non-uniform) abatement factors.[2] As argued by Kousmanen (2005), the conventional specification of WD technology based on uniform abatement factor goes against the usual observed operating practices of focusing abatement efforts in those firms where marginal abatement costs are lowest. To circumvent this, he proposed a more general specification of WD technology that allows for non-uniform abatement factors depending upon the degree of the marginal abatement costs by the firms. Both the specifications of WD technology have been critically debated with pros and cons.[3]

Though Kousmanen's formulation of WD environmental technology seems theoretically more promising, a study by Sahoo et al. (2011) on the environmental efficiency (EE) performance of 22 OECD (The Organisation for Economic Cooperation and Development) countries reveals that both the formulations of the WD technology yield identical EE estimates across all the countries.[4] The occurrence of such unexpected finding reported in their empirical application questions the theoretical advantage of one model over the other. In this paper an attempt is, therefore, made to show that when the efficiency projection of an inefficient firm does not lie on the WD frontier region of the environmental

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technology, both the formulations of WD technology yield identical EE estimates; and when the efficiency projection is made onto the WD region of the frontier, both the formulations yield differential EE estimates.

The remainder of this paper proceeds as follows. Section 2 deals with the description of the two alternative formulations of WD environmental technology, and their resultant underlying EE measures. An empirical illustration is made in Section 3 to show, when both the formulations of WD environmental technology yield identical EE estimates; and when they yield differential EE estimates.

DEA Environmental Technology

The WD environmental technology structure with a constant uniform abatement factor (ϕ) across firms[5] is set up as:

$$T^{UA}=\left\{\begin{array}{l}\left(y^g,y^b,x\right):\sum\limits_{j\in I_N} y^g_{rj}\phi\lambda_j \geq y^g_r\ (\forall r\in I_G),\sum\limits_{j\in I_N} y^b_{rj}\phi\lambda_j = y^b_r\ (\forall r\in I_B),\\ \sum\limits_{j\in I_N} x_{ij}\lambda_j \leq x_i(\forall i\in I_M),\ \sum\limits_{j\in I_N}\lambda_j=1,\ \lambda_j\geq 0\ (\forall j\in I_N),\ 0\leq\phi\leq 1\end{array}\right\} \tag{1}$$

Note that T^{UA} is non-linear in construct and therefore needs linearisation. The linear equivalent structure of T^{UA} is set up as:

$$T^{UA(L)}=\left\{\begin{array}{l}\left(y^g,y^b,x\right):\sum\limits_{j\in I_N} y^g_{rj}\lambda'_j \geq y^g_r(\forall r\in I_G),\sum\limits_{j\in I_N} y^b_{rj}\lambda'_j = y^b_r(\forall r\in I_B),\\ \sum\limits_{j\in I_N} x_{ij}\lambda'_j \leq \phi x_i\ (\forall i\in I_M),\sum\limits_{j\in I_N}\lambda'_j=\phi,\phi\leq 1,\lambda'_j\geq 0\ (\forall j\ \in\ I_N)\end{array}\right\} \tag{1a}$$

where $\lambda'_j=\phi\lambda_j$.

However, as pointed out by Kousmanen (2005), accounting for weak disposability with a uniform abatement factor ϕ across firms in T^{UA} is very restrictive, which goes against the very usual observed practice of focusing abatement efforts in those firms where the marginal abatement costs are lowest. Therefore, he suggested an alternative formulation of weak disposability that accounts for non-uniform (differential) abatement factors (ϕ_j) across firms depending upon the degree of their marginal abatement costs. Therefore, on substitution ϕ of by ϕ_j in ($0\leq\phi_j\leq 1$) in T^{UA} yields an alternative WD technology structure, which is represented as:

$$T^{DA}=\left\{\begin{array}{l}\left(y^g,y^b,x\right):\sum\limits_{j\in I_N} y^g_{rj}\phi_j\lambda_j \geq y^g_r\ (\forall r\in I_G),\sum\limits_{j\in I_N} y^b_{rj}\phi_j\lambda_j = y^b_r\ (\forall r\in I_B),\\ \sum\limits_{j\in I_N} x_{ij}\lambda_j \leq x_i(\forall i\in I_M),\ \sum\limits_{j\in I_N}\lambda_j=1,\ \lambda_j\geq 0\ (\forall j\in I_N),\ 0\leq\phi_j\leq 1\ (\forall j)\end{array}\right\} \tag{2}$$

In order to linearize T^{DA} he breaks down the intensity weight of firm *j* (λ_j) into two parts, i.e., $\lambda_j = \psi_j + \mu_j$. The first part (ψ_j) represents that part of the output, which remains active, i.e., $\psi_j = \phi_j \lambda_j$ and the second part (μ_j) represents that part of the output, which is abated through scaling down of activity level, i.e., $\mu_j = (1-\phi_j)\lambda_j$. Here the abatement scaling factor (ϕ_j) is defined as $\phi_j = \psi_j / (\psi_j + \mu_j)$. On substitution of these notations in T^{DA} yields the following linear technology structure:

$$T^{DA(L)} = \left\{ \begin{array}{l} \left(y^g, y^b, x\right) : \sum_{j \in I_N} y^g_{rj} \psi_j \geq y^g_r \; (\forall r \in I_G), \sum_{j \in I_N} y^b_{rj} \psi_j = y^b_r \; (\forall r \in I_B), \\ \sum_{j \in I_N} x_{ij} (\psi_j + \mu_j) \leq x_i (\forall i \in I_M), \sum_{j \in I_N} (\psi_j + \mu_j) = 1, \; \psi_j, \mu_j \geq 0 \; (\forall j \in I_N) \end{array} \right\} \quad (2a)$$

Based on the WD environmental technology ($T^{UA(L)}$), we consider the following EE measure of firm *h* (EE^{UA}_h) that simultaneously maximizes y^g and minimizes y^b by keeping inputs fixed:

$$\left[WD^{UA}\right] \quad \left(1 - EE^{UA}_h\right) = \max \left\{\theta : \left(y^g_h + \theta z^g, y^b_h - \theta z^b, x_h\right) \in T^{UA(L)}\right\} \quad (3)$$

Similarly, based on the WD environmental technology ($T^{DA(L)}$), the corresponding EE measure of firm *h* (EE^{DA}_h) can be set up as:

$$\left[WD^{DA}\right] \quad \left(1 - EE^{DA}_h\right) = \max \left\{\theta : \left(y^g_h + \theta z^g, y^g_h - \theta z^b, x_h\right) \in T^{DA(L)}\right\} \quad (4)$$

In both (3) and (4), the output direction vector is $z = \left(z^g, z^b\right)$ where z^g and z^b elements indicate, respectively, the expansion and contraction paths for desirable and undesirable outputs.

Note that $T^{UA(L)} = T^{DA(L)}$ when $x_h = k$, $(\forall h \in I_N)$ where *k* is some constant; and the resultant EE scores obtained from [WD^{UA}] and [WD^{DA}] are the same, i.e., $EE^{UA}_h = EE^{DA}_h$.

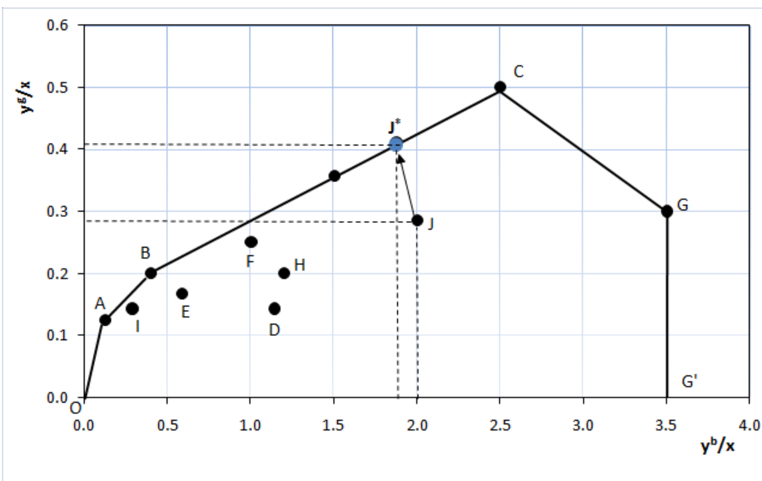
An Empirical Illustration

Consider a simple hypothetical data set exhibited in Table 1 consisting of 10 firms labeled as A, B, C, D, E, F, G, H, I and J. Each firm is assumed to produce two outputs – one desirable output and one undesirable output using one input. Based on these data, we exhibit in Figure 1 the environmental technology set in the form of output set in which we illustrate the measure of EE.

Table 1: An Example Data set with EE Scores

Firms	*Input*	*Good output*	*Bad output*	*EE scores*	
	x	y^g	y^b	WD^{UA}	WD^{DA}
A	16	2	2	1	1
B	15	3	6	1	1
C	3	1.5	7.5	1	1
D	28	4	32	0.857	0.857
E	12	2	7	0.948	0.948
F	24	6	24	0.969	0.969
G	4	1.2	14	1	1
H	40	8	48	0.9	0.9
I	35	5	10	0.980	0.980
J	14	4	28	0.875	0.875

Source: Sahoo et al. (2011, p. 755)



Source: Sahoo et al., 2011, p. 755.

Figure 1: Environmental Technology

Under the specification of WD technology based on uniform abatement factor, OABCGG' forms the regulated technology frontier where four firms – A, B, C and G are found efficient while others are inefficient. As to measuring EE, both [WD^{UA}] and [WD^{DA}] measures project, for example, an inefficient firm J to point J* where EE_J=1–θ 1–0.125=0.875, using the output direction vector $z=(z^g,–z^b)$ as (1,-1)).

Though both the models – [WD^{UA}] and [WD^{DA}] are structurally different with respect to the abatement factor (ϕ), the EE scores obtained from these models are same (See Table 1). This finding is not strange because since all the firms use the same unity resource input, the corresponding abatement factors ϕ_j of all the firms are all equal in which case the technology structures in [WD^{II}] and [WD^{II*}] models are essentially the same.

Now we turn to illustrate both the measures of EE using the OECD data reported in Sahoo et al.[6] Surprisingly, both the models exhibit the same EE estimates for all the countries (See Table 2). We $\phi = 1$ find for all the 22 countries even though the input resources – labor and capital are not the same across the countries. We conjectured that this unexpected finding might be due to the possibility that the positions of all the inefficient countries are such that their projections did not lie on the WD frontier region of the environmental technology, which appears quite strange in a real-life data set like this.

Table 2: EE Scores Based on OECD Data Set

	1995		*2004*	
Countries	WD^{UA}	WD^{DA}	WD^{UA}	WD^{DA}
Austria	0.943	0.943	0.908	0.908
Belgium	0.998	0.998	0.932	0.932
Denmark	0.878	0.878	0.801	0.801
Finland	0.798	0.798	0.741	0.741
France	1	1	1	1
Germany	0.918	0.918	0.925	0.925
Greece	0.653	0.653	0.752	0.752
Ireland	1	1	1	1
Italy	1	1	0.997	0.997
Netherlands	0.903	0.903	0.864	0.864
Portugal	1	1	0.904	0.904
Spain	0.969	0.969	0.922	0.922
Sweden	1	1	1	1
United Kingdom	1	1	1	1
Australia	0.887	0.887	1	1
Canada	0.903	0.903	0.909	0.909
Iceland	1	1	1	1
Japan	1	1	1	1
New Zealand	0.771	0.771	0.638	0.638
Norway	1	1	1	1
Switzerland	1	1	1	1
United States	1	1	1	1

To empirically demonstrate the conditions for which [WD^{UA}] and [WD^{DA}] yield identical EE estimates, and the conditions for which [WD^{UA}] and [WD^{DA}] yield differential EE, for a firm, we consider an example data set where each firm uses one input to produce one desirable output and one undesirable output. Based on this data, we draw the eco-efficiency frontier in Figure 2 using the two formulations of WD technology – one based on the uniform abatement factor and the other on the non-uniform abatement factors. Note that this figure is taken from Kousmanen (2005)[7] in which we have introduced two more new firms – D and E.

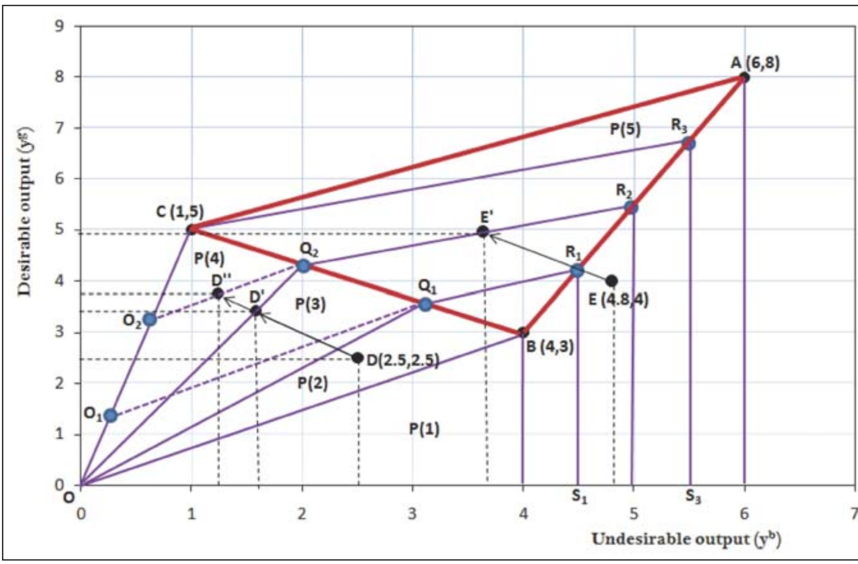


Figure 2: Environmental Technology Structures with WD^{UA} and WD^{DA}

Five firms are labeled as A, B, C, D and E whose input-output vectors are (5,8,6), (1,3,4), (4,5,1), (3,2.5,2.5) and (3,4,4.8) respectively. Here, the first, second and third term in each bracket represents, respectively, the input, the bad output and the good output. Under the assumption of uniform abatement factor, the output sets at various levels of inputs are: P(1) = the triangle with vertices O, B and (4,0); P(2) = the quadrilateral with vertices O, Q_1, R_1, and S_1; P(3) = the quadrilateral with vertices O, Q_2, R_2, and (5,0); P(4) = the quadrilateral with vertices O, C, R_3, and S_3; and P(5) = the quadrilateral with vertices O, C, A, and (6,0). The output isoquants for the various levels of inputs are represented by the boundaries of the respective output sets. However, under the assumption of non-uniform abatement factors, the output sets – P(1), P(4) and P(5) remain the same, but the output sets – P(2) and P(3) become the pentagons – with vertices O, O_1, Q_1, R_1 and S_1 for P(2); and O, O_2, Q_2, R_2 and (5,0) for P(3).

We now consider the EE evaluation of firm D, which operates on the WD region of the environmental technology set. Under the assumption of uniform abatement factor, D can be projected to point D′ of the isoquant segment OQ_2 of the boundary of P(3) whose co-ordinates are (1.579,3.421) that are obtained from the solutions of $[WD^{UA}]$ in (3): θ = 0.3684, ϕ = 0.7895, λ'_B = 0.2632 and λ'_C = 0.5263. And, under the assumption of differential abatement factors, D can be projected to point D" of the isoquant segment O_2Q_2 of the boundary of P(3) whose co-ordinates are (1.238.3.762), which are obtained from the solutions of $[WD^{DA}]$ in (4): θ = 0.5048, ψ_B = 0.1429, ψ_C = 0.6667 and μ_B = 0.1905. However, in case of firm E, which operates not on the WD region of the technology, both $[WD^{UA}]$ and $[WD^{DA}]$ project it onto the same point, i.e., E (3.636,4.9 70) of the isoquant segment Q_2R_2 of the boundary of P(3). $[WD^{UA}]$ yields: θ = 0.2424, ϕ = 1, $=\lambda'_A$ 0.2727, λ'_B = 0.4242 and λ'_C = 0.3030 and $[WD^{DA}]$ yields: θ = 0.2424, ψ_A = 0.2727, ψ_B = 0.4242 and ψ_C = 0.3030.

Therefore, it can be concluded that whenever an inefficient firm lies on the WD facet OBC of environmental technology, the EE scores obtained From $[WD^{DA}]$ is no more than those of $[WD^{UA}]$. However, if a firm operates on any other facet of the technology, both the formulations of environmental technology yield identical EE estimates. Therefore, the question remains to be answered as to the choice of the empirical use of either measure in revealing proper EE behavior of firms as our real-life empirical application suggests that both $[WD^{DA}]$ and $[WD^{UA}]$ yields identical EE estimates.

Notes

1. R.Färe, S. Grosskopf, "Nonparametric productivity analysis with undesirable outputs: Comment", *American Journal of Agricultural Economics*, No. 85, 2003, pp. 1070–1074.
2. T. Kuosmanen, "Weak disposability in nonparametric production analysis with undesirable outputs", *American Journal of Agricultural Economics*, No. 87, 2005, pp. 1077–1082.
3. R. Färe, S. Grosskopf, "A comment on weak disposability in nonparametric production analysis", *American Journal of Agricultural Economics*, No. 91, 2009, pp. 535–538; T. Kuosmanen, V. Podinovski, "Weak disposability in nonparametric production analysis: Reply to Fare and Grosskopf", *American Journal of Agricultural Economics*, No. 91, 2009, pp. 539–545; V. Podinovski, T. Kousmanen, "Modelling weakly

disposability in data envelopment analysis under relaxed convexity assumption", *European Journal of Operational Research*, No. 211, 2011, pp. 577–585.

4. B.K. Sahoo, M. Luptacik, B. Mahlberg, "Alternative measures of environmental technology structure in DEA: An application", *European Journal of Operational Research*, No. 215, 2011, pp. 750–762.
5. R. Färe, S. Grosskopf, "Nonparametric productivity analysis with undesirable outputs: Comment", *American Journal of Agricultural Economics*, No. 85, 2003, pp. 1070–1074; R. Färe, S. Grosskopf, "Modeling undesirable factors in efficiency evaluation: Comment", *European Journal of Operational Research*, No. 157, 2004, pp. 242–245.
6. Sahoo et al., n. 4, p. 759.
7. Kuosmanen, n. 2, p. 1081.